Reg. No. : [ ][ ][ ][ ][ ][ ][ ][ ][ ][ ][ ][ ]

Question Paper Code : X 60764

B.E./B.Tech. DEGREE EXAMINATIONS, NOV./DEC. 2020
First Semester
Civil Engineering
MA 2111/MA12/080030001 – MATHEMATICS – I
(Common to All Branches)
(Regulations 2008)

Time : Three Hours Maximum : 100 Marks.

Answer ALL questions.

PART – A **(10×2=20 Marks)**

1. If $A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 4 & 5 \\ 0 & 0 & 6 \end{bmatrix}$ then find the eigen values of A^{-1}.

2. State Cayley-Hamilton theorem.

3. Find the centre and radius of the sphere $2x^2 + 2y^2 + 2z^2 + 6x - 6y + 8z + 9 = 0$.

4. Prove that the equation $x^2 - 2y^2 + 3z^2 + 5yz - 6zx - 4xy + 8x - 19y - 2z - 20 = 0$ represents a cone with vertex $(1, -2, 3)$.

5. Find the curvature of the circle $x^2 + y^2 = 25$ at the point $(4, 3)$.

6. Define evolute of the curve.

7. If $u = \frac{x}{y} + \frac{y}{z} + \frac{z}{x}$, show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z} = 0$.

8. If $u = \frac{y^2}{2x}, v = \frac{x^2 + y^2}{2x}$, find $\frac{\partial(u, v)}{\partial(x, y)}$.

9. Plot the region of integration to evaluate the integral $\iint_D f(x, y)\, dx\, dy$ where D is the region bounded by the line $y = x - 1$ and the parabola $y^2 = 2x + 6$.

10. Evaluate $\int_0^2 \int_0^\pi r \sin^2 \theta \, d\theta dr$.

PART – B **(5×16= 80 Marks)**

11. a) i) Verify Cayley-Hamilton theorem for a matrix $A = \begin{bmatrix} 1 & 0 & 3 \\ 2 & 1 & -1 \\ 1 & -1 & 1 \end{bmatrix}$ and hence find A^{-1}. **(8)**

ii) Find the eigen values and eigenvectors of a matrix $\begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$. **(8)**

(OR)

b) Reduce the quadratic form $8x^2 + 7y^2 + 3z^2 - 12xy - 8yz + 4zx$ to the canonical form through an orthogonal transformation. Hence find the following :

i) Nature of the quadratic form.

ii) Rank, index and signature of the quadratic form and

iii) A set of non-zero values of x, y, z which will make the quadratic form zero. **(16)**

12. a) i) Find the equations of the tangent planes to the sphere $x^2 + y^2 + z^2 - 4x - 2y + 6z + 5 = 0$ which are parallel to the plane $x + 4y + 8z = 0$. Find also their points of contact . **(8)**

ii) Find the equation of the right circular cone whose vertex is (2, 1, 0), semivertical angle is 30° and the axis is the line $\frac{x-2}{3} = \frac{y-1}{1} = \frac{z}{2}$. **(8)**

(OR)

b) i) Find the equation of the cylinder whose generators are parallel to $\frac{x}{2} = \frac{y}{2} = \frac{z}{-3}$ and whose guiding curve is the ellipse $3x^2 + y^3 = 3, z = 2$. **(8)**

ii) Show that the plane $2x - 2y + z + 12 = 0$ touches the sphere $x^2 + y^2 + z^2 - 2x - 4y + 2z = 3$ and also find the point of contact. **(8)**

13. a) i) Prove that for the curve $y = \frac{ax}{a+x}, \left(\frac{2p}{a}\right)^{\frac{2}{3}} = \left(\frac{x}{y}\right)^2 + \left(\frac{y}{x}\right)^2$. **(8)**

ii) Find the envelope of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ where a and b are connected by the relation $a^2 + b^2 = c^2$, c being a constant. **(8)**

(OR)

b) i) Obtain the equation of the evolute of the curve $x = a(\cos\theta + \theta\sin\theta)$, $y = a(\sin\theta - \theta\cos\theta)$. **(8)**

ii) Prove that the radius of curvature of the curve $xy^2 = a^3 - x^3$ at the point (a, 0) is $\frac{3a}{2}$. **(8)**

14. a) i) If $u = \tan^{-1}\left[\frac{x+y}{\sqrt{x}+\sqrt{y}}\right]$ show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \frac{1}{4}\sin 2u$. **(8)**

ii) Find the Jacobian of y_1, y_2, y_3 with respect to x_1, x_2, x_3 if

$y_1 = \frac{x_2 x_3}{x_1}, y_2 = \frac{x_3 x_1}{x_2}, y_3 = \frac{x_1 x_2}{x_3}$. **(8)**

(OR)

b) i) Expand $\tan^{-1}\left(\frac{y}{x}\right)$ as a Taylor series about the point (1, 1) upto 2^{nd} degree terms. **(8)**

ii) Find the shortest distance from the point (1, 0) to the parabola $y^2 = 4x$. **(8)**

15. a) i) Change the order of integration $\int_0^1 \int_{x^2}^{2-x} xy\,dy\,dx$ and hence evaluate. **(8)**

ii) Transform the integral into polar coordinates and hence evaluate

$\int_0^a \int_0^{\sqrt{a^2-x^2}} \sqrt{x^2+y^2}\,dy\,dx$. **(8)**

(OR)

b) i) Find, by double integration, the area between the two parabolas $3y^2 = 25x$ and $5x^2 = 9y$. **(8)**

ii) Find the volume of the portion of the cylinder $x^2 + y^2 = 1$ intercepted between the plane x = 0 and the paraboloid $x^2 + y^2 = 4 - z$. **(8)**